# Prealgebra Textbook

Second Edition

## Chapter 8

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# Contents

*Chapter 8*

# Graphing

René Descartes (1596-1650) was a French philosopher and mathematician. As a philosopher, he is famous for the saying “Cogito ergo sum” (“I think, therefore I am”), and his writings led many to consider him the *Father of Modern Philosophy*. Even today, a number of his writings are standard faire in university philosophy departments.

However, it is Descartes’ work in mathematics that form the basis for this chapter, particularly his invention of the *Cartesian Coordinate System* which bears his name. Descartes’ invention of the coordinate system created an entirely new branch of mathematics called *analytic geometry*, which established a permanent link between the plane and solid geometry of the ancient Greeks and the algebra and analysis of modern mathematics. As a result of his work, mathematicians were able to describe curves with equations, unheard of before Descartes’ invention of the coordinate system. Rather than describing a circle as the “locus of all points equidistant from a given point,” mathematicians were now able to refer to a circle centered at the point $(0, 0)$ with radius $r$ as the graph of the equation $x^2 + y^2 = r^2$.

The bridge created between geometry and analysis as a result of Descartes’ methods laid the groundwork for the discovery of the calculus by Newton and Leibniz. For his efforts, mathematicians often refer to Descartes as the *Father of Analytic Geometry*.

In this chapter we will introduce readers to the Cartesian coordinate system and explain the correspondence between points in the plane and ordered pairs of numbers. Once an understanding of the coordinate system is sufficiently developed, we will develop the concept of the graph of an equation. In particular, we will address the graphs of a class of equations called *linear equations*.

## 8.1 The Cartesian Coordinate System

Let's begin with the concept of an *ordered pair* of whole numbers.

**Ordered Pairs of Whole Numbers.** The construct $(x, y)$, where $x$ and $y$ are whole numbers, is called an ordered pair of whole numbers.

Examples of ordered pairs of whole numbers are $(0, 0)$, $(2, 3)$, $(5, 1)$, and $(4, 9)$.

**Order Matters.** Pay particular attention to the phrase "ordered pairs." Order matters. Consequently, the ordered pair $(x, y)$ is not the same as the ordered pair $(y, x)$, because the numbers are presented in a different order.

We've seen how to plot whole numbers on a number line. For example, in Figure 8.1, we've plotted the whole numbers 2, 5, and 7 as shaded "dots" on the number line.

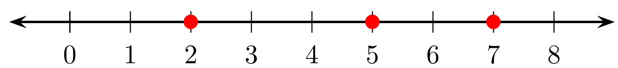
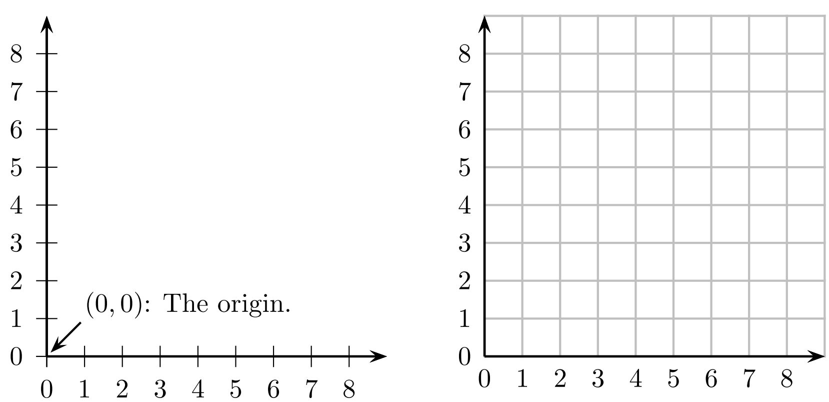



Figure 8.1: Plotting the whole numbers 2, 5, and 7 on a number line.

To plot ordered pairs, we need two number lines, called the *horizontal and vertical axes*, that intersect at the zero location of each line and are at right angles to one another, as shown in Figure 8.2(a). The point where the zero locations touch is called the *origin* of the coordinate system and has coordinates $(0, 0)$. In Figure 8.2(b), we've added a grid. The resulting construct is an example of a *Cartesian Coordinate System.*

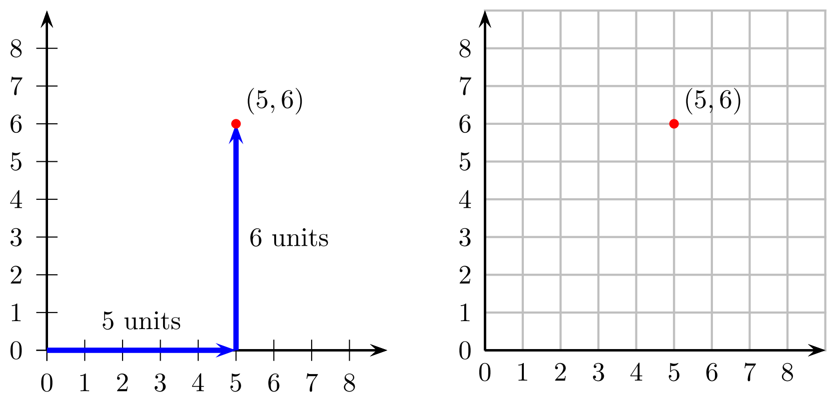



(a) Horizontal and vertical axes.

(b) Adding a grid.

Figure 8.2: A Cartesian coordinate system.

Now, consider the ordered pair of whole numbers $(5, 6)$. To plot this point on the "coordinate system" in Figure 8.3(a), start at the origin $(0, 0)$, then move 5 units in the horizontal direction, then 6 units in the vertical direction, then plot a point. The result is shown in Figure 8.3(a). Adding a grid of horizontal and vertical lines at each whole number makes plotting the point $(5, 6)$ much clearer, as shown in Figure 8.3(b).




(a) 5 right and 6 up. (b) Plotting on a grid.

Figure 8.3: Plotting the Point $(5, 6)$ in a Cartesian Coordinate System.

The numbers in the ordered pair $(5, 6)$ are called the *coordinates* of the plotted point in Figure 8.3(b). The first number of the ordered pair is called the *abscissa* and measures the horizontal distance to the plotted point. The second number is called the *ordinate* and measures the vertical distance to the plotted point. The combination of axes and grid in Figure 8.3(b) is called a *coordinate system*.

The grid in Figure 8.3(b) is a visualization that greatly eases the plotting of ordered pairs. However, you don't have to draw these gridlines yourself. Instead, you should work on graph paper.

**Graph Paper Requirement.** All plotting should be done on graph paper.

**You Try It!**

**EXAMPLE 1.** Plot the following ordered pairs of whole numbers: $(3, 2)$, $(8, 6)$, and $(2, 7)$.

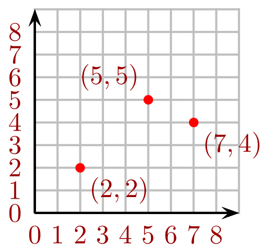
Plot the following ordered pairs of whole numbers: $(2, 2)$, $(5, 5)$, and $(7, 4)$.

**Solution.** Create a Cartesian coordinate system on graph paper, then:

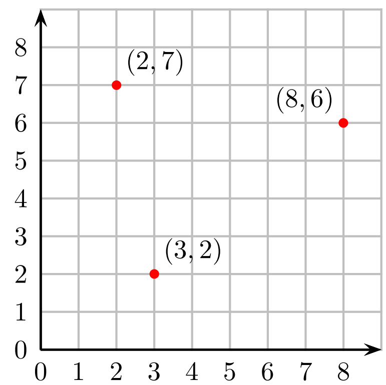
- To plot the ordered pair $(3, 2)$, start at the origin, then move 3 units to the right and 2 units up.

- To plot the ordered pair $(8, 6)$, start at the origin, then move 8 units to the right and 6 units up.
- To plot the ordered pair $(2, 7)$, start at the origin, then move 2 units to the right and 7 units up.

Answer:




The results are shown on the following Cartesian coordinate system.




## Allowing for Negative Numbers

Again, we've seen how to plot both positive and negative numbers on a number line. For example, in Figure 8.4, we've plotted the numbers $-4$, $-3/2$, 2.2 and 4.

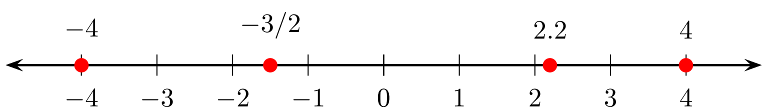
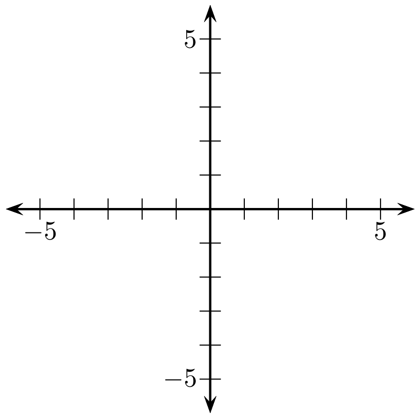


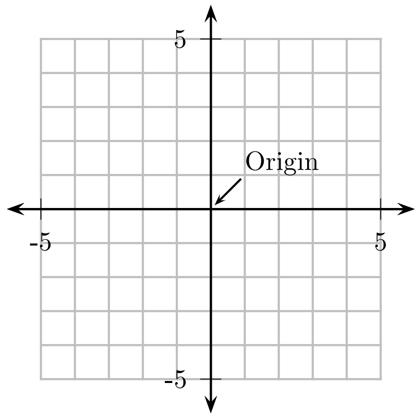

Figure 8.4: Plotting the numbers $-4$, $-3/2$, 2.2, and 4.

Note that the positive direction is to the right, the negative to the left. That is, to plot the number 2.2, we move 2.2 units to the right on the line, but to plot the number $-3/2$, we move $3/2$ units to the left.

To plot ordered pairs having both positive and negative numbers, we need two such number lines that intersect at the zero location of each line and are at right angles to one another, as shown in Figure 8.5(a). As before, adding a grid of horizontal and vertical lines at each integer will be extremely helpful when plotting points (see Figure 8.5(b)). The system of axes and grid in Figure 8.5(b) is called the *Cartesian Coordinate System*, named after its inventor, Renè Descartes.




(a) Horizontal and vertical axes.




(b) Adding a grid.

Figure 8.5: The Cartesian coordinate system.

**Plotting Points in the Cartesian Coordinate System.** On the horizontal axis, the positive direction is to the right, negative is to the left. On the vertical axis, the positive direction is up, negative is down. The point $(0, 0)$ is called the *origin* of the coordinate system, and is the starting point for all point plotting.

**You Try It!**

Sketch the points $(3, 4)$, $(-4, 3)$, $(-3, -4)$, and $(4, -3)$ on a Cartesian coordinate system.

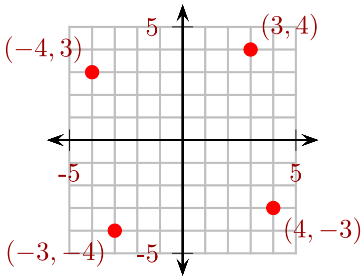
**EXAMPLE 2.** Sketch the points $(4, 3)$, $(-3, 2)$, $(-2, -4)$, and $(3, -3)$ on a Cartesian coordinate system.

**Solution.** Set up a Cartesian coordinate system on graph paper.

- To plot the point $(4, 3)$, start at the origin, move 4 units to the right, then 3 units up.
- To plot the point $(-3, 2)$, start at the origin, move 3 units to the left, then 2 units up.
- To plot the point $(-2, -4)$, start at the origin, move 2 units to the left, then 4 units down.
- To plot the point $(3, -3)$, start at the origin, move 3 units to the right, then 3 units down.

These points are plotted and shown in Figure 8.6.

Answer:



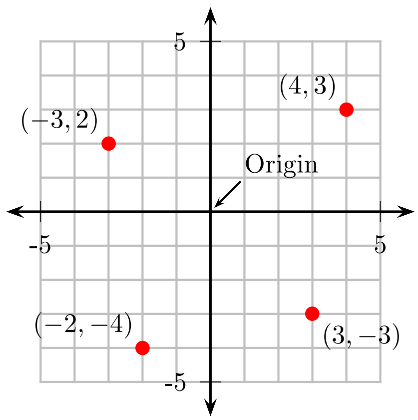




Figure 8.6: Plotting points in the Cartesian coordinate system.

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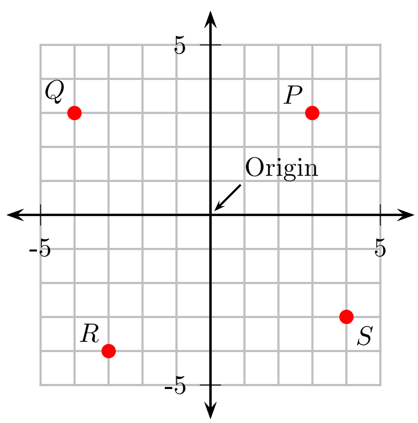
**You Try It!**

**EXAMPLE 3.** What are the coordinates of the points $P$, $Q$, $R$, and $S$ in the Cartesian coordinate system that follows?

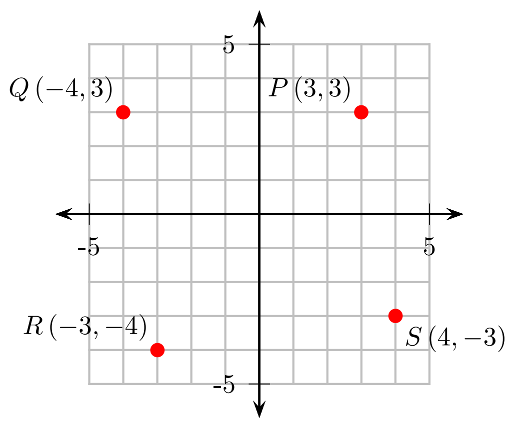



**Solution.** Make all measurements from the origin.

- To obtain the coordinates of point $P$, start at the origin, move 3 units to the right, then 3 units up. Hence, the coordinates of the point $P$ are $(3, 3)$.

- To obtain the coordinates of point $Q$, start at the origin, move 4 units to the left, then 3 units up. Hence, the coordinates of the point $Q$ are $(-4, 3)$.
- To obtain the coordinates of point $R$, start at the origin, move 3 units to the left, then 4 units down. Hence, the coordinates of the point $R$ are $(-3, -4)$.
- To obtain the coordinates of point $S$, start at the origin, move 4 units to the right, then 3 units down. Hence, the coordinates of the point $S$ are $(4, -3)$.

These results are shown on the following Cartesian coordinate system.

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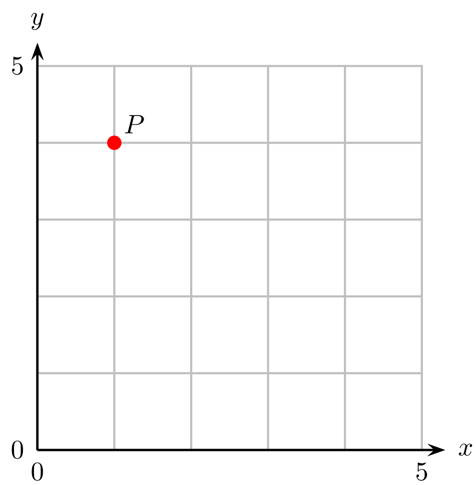
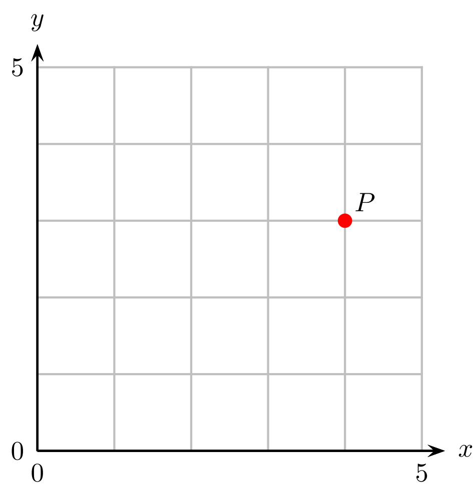
## ❧ ❧ ❧ Exercises ☙ ☙ ☙

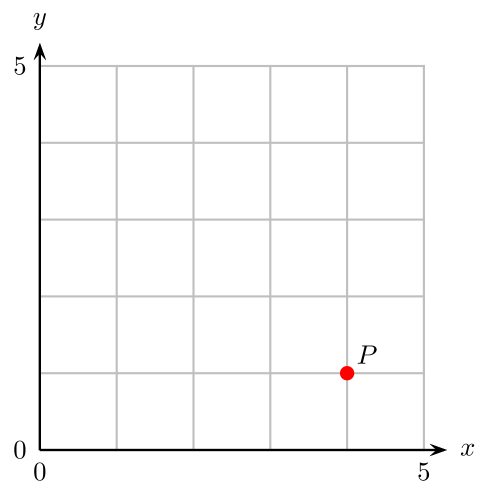
**1**. Identify the coordinates of the point $P$.



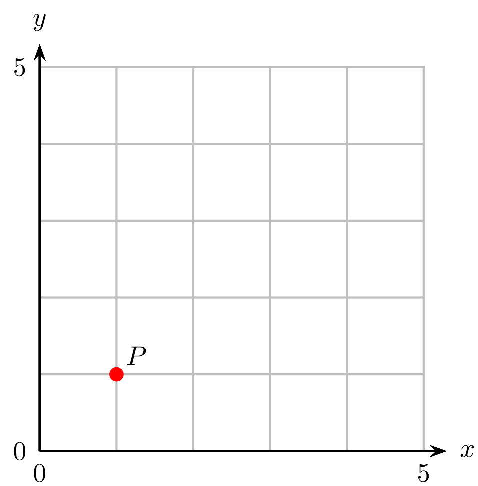

**2**. Identify the coordinates of the point $P$.



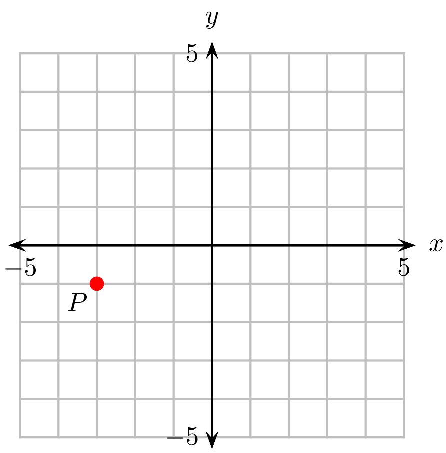

**3**. Identify the coordinates of the point $P$.



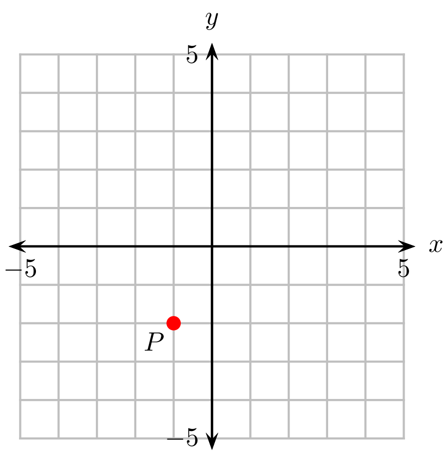

**4**. Identify the coordinates of the point $P$.



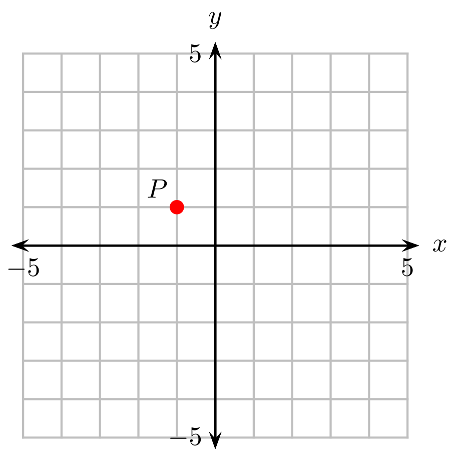
**5**. Identify the coordinates of the point $P$.



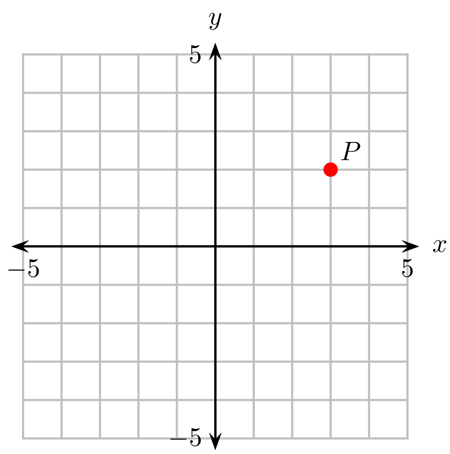

**6**. Identify the coordinates of the point $P$.




**7**. Identify the coordinates of the point $P$.




**8**. Identify the coordinates of the point $P$.

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**9**. The points $A(-1, 1)$, $B(1, 1)$, $C(1, 2)$, and $D(-1, 2)$ are the vertices of a rectangle. Plot these points, draw the rectangle $ABCD$, then compute the area of rectangle $ABCD$.

**10**. The points $A(-3, -4)$, $B(4, -4)$, $C(4, -1)$, and $D(-3, -1)$ are the vertices of a rectangle. Plot these points, draw the rectangle $ABCD$, then compute the area of rectangle $ABCD$.

**11**. The points $A(-2,-1)$, $B(3,-1)$, $C(3,3)$, and $D(-2,3)$ are the vertices of a rectangle. Plot these points, draw the rectangle $ABCD$, then compute the area of rectangle $ABCD$.

**12**. The points $A(-3,-1)$, $B(2,-1)$, $C(2,2)$, and $D(-3,2)$ are the vertices of a rectangle. Plot these points, draw the rectangle $ABCD$, then compute the area of rectangle $ABCD$.

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**13**. The points $A(-4,-2)$, $B(1,-2)$, $C(1,1)$, and $D(-4,1)$ are the vertices of a rectangle. Plot these points, draw the rectangle $ABCD$, then compute the perimeter of rectangle $ABCD$.

**14**. The points $A(-4,-4)$, $B(1,-4)$, $C(1,-3)$, and $D(-4,-3)$ are the vertices of a rectangle. Plot these points, draw the rectangle $ABCD$, then compute the perimeter of rectangle $ABCD$.

**15**. The points $A(-1,2)$, $B(3,2)$, $C(3,3)$, and $D(-1,3)$ are the vertices of a rectangle. Plot these points, draw the rectangle $ABCD$, then compute the perimeter of rectangle $ABCD$.

**16**. The points $A(-4,2)$, $B(3,2)$, $C(3,4)$, and $D(-4,4)$ are the vertices of a rectangle. Plot these points, draw the rectangle $ABCD$, then compute the perimeter of rectangle $ABCD$.

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**17**. The points $A(-3,-1)$, $B(1,-1)$, and $C(-3,0)$ are the vertices of a triangle. Plot these points, draw the triangle $ABC$, then compute the area of the triangle $ABC$.

**18**. The points $A(-3,-2)$, $B(1,-2)$, and $C(-3,2)$ are the vertices of a triangle. Plot these points, draw the triangle $ABC$, then compute the area of the triangle $ABC$.

**19**. The points $A(-1,-2)$, $B(0,-2)$, and $C(-1,0)$ are the vertices of a triangle. Plot these points, draw the triangle $ABC$, then compute the area of the triangle $ABC$.

**20**. The points $A(-2,-3)$, $B(-1,-3)$, and $C(-2,1)$ are the vertices of a triangle. Plot these points, draw the triangle $ABC$, then compute the area of the triangle $ABC$.

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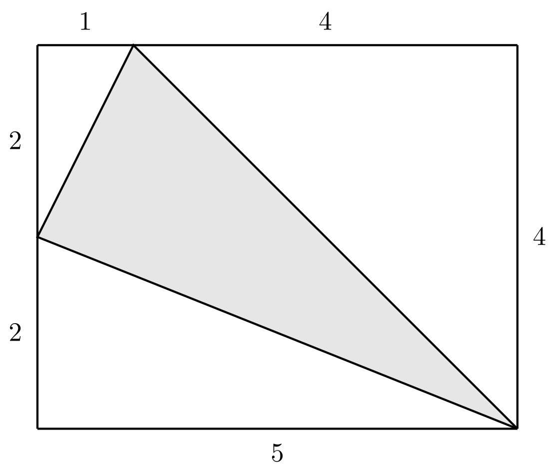
**21**. Plot the points $A(-3,-3)$ and $B(0,0)$ and find the straight-line distance between the two points. *Hint: Create a right triangle, then use the Pythagorean Theorem.*

**22**. Plot the points $A(-2,-3)$ and $B(1,2)$ and find the straight-line distance between the two points. *Hint: Create a right triangle, then use the Pythagorean Theorem.*

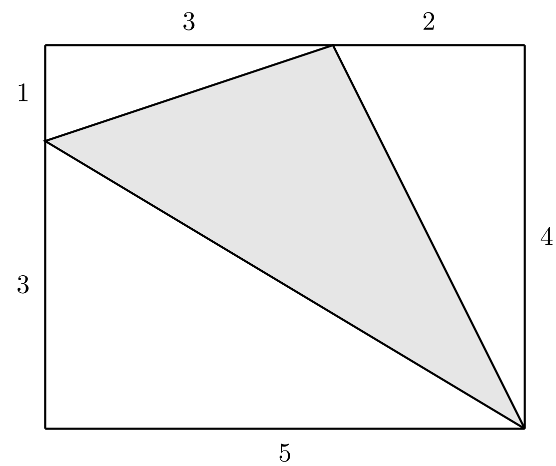
**23**. Plot the points $A(-2,-3)$ and $B(0,0)$ and find the straight-line distance between the two points. *Hint: Create a right triangle, then use the Pythagorean Theorem.*

**24**. Plot the points $A(-3,-2)$ and $B(2,2)$ and find the straight-line distance between the two points. *Hint: Create a right triangle, then use the Pythagorean Theorem.*

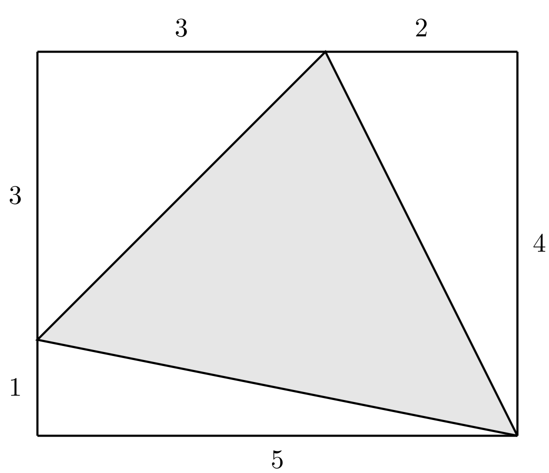
**25**. Find the area of the shaded triangle.



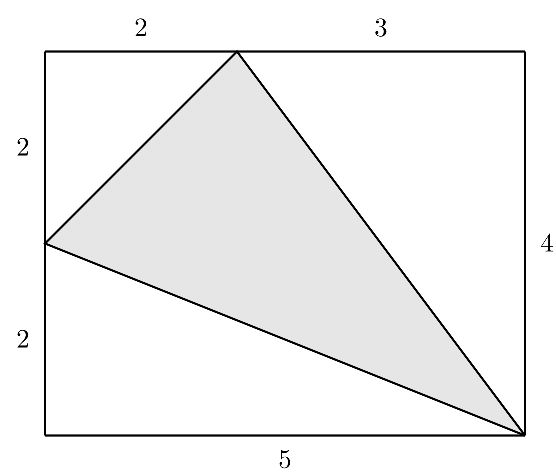

**27**. Find the area of the shaded triangle.




**26**. Find the area of the shaded triangle.




**28**. Find the area of the shaded triangle.



**Hint:** In Exercises 29-32, surround the triangle with a rectangle, like those shown in Exercises 25-28.

**29**. Find the area of the triangle with vertices at $A(-4,-1)$, $B(4,-2)$, and $C(1,3)$.

**30**. Find the area of the triangle with vertices at $A(-4,2)$, $B(3,0)$, and $C(0,4)$.

**31**. Find the area of the triangle with vertices at $A(-3,1)$, $B(3,-3)$, and $C(1,4)$.

**32**. Find the area of the triangle with vertices at $A(1,2)$, $B(3,0)$, and $C(2,4)$.

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## ❧ ❧ ❧ Answers ☙ ☙ ☙

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**1.** $(1,4)$

**3.** $(4,1)$

**5.** $(-3,-1)$

**7.** $(-1,1)$

**9.** 2 square units

**11.** 20 square units

**13.** 16 units

**15.** 10 units

**17.** 2 square units

**19.** 1 square units

**21.** $\sqrt{18}$

**23.** $\sqrt{13}$

**25.** 6

**27.** 7

**29.** $\dfrac{37}{2}$

**31.** 17

## 8.2 Graphing Linear Equations

Consider $y = x + 1$ an *equation in two variables.* If we substitute the ordered pair $(x, y) = (1, 2)$ into the equation $y = x + 1$, that is, if we replace $x$ with 1 and $y$ with 2, we get a true statement.

| | |
|---|---|
| $y = x + 1$ | Original equation. |
| $2 = 1 + 1$ | Substitute: 1 for $x$ and 2 for $y$. |
| $2 = 2$ | Simplify. |

We say that the ordered pair $(1, 2)$ is a *solution* of the equation $y = x + 1$.

**Solution of an Equation in Two Variables.** If substituting the ordered pair $(x, y) = (a, b)$ into an equation (replace $x$ with $a$ and $y$ with $b$) produces a true statement, then the ordered pair $(a, b)$ is called a *solution* of the equation and is said to "satisfy the equation."

**You Try It!**

**EXAMPLE 1.** Which of the ordered pairs are solutions of the equation $y = 2x + 5$: (a) $(-3, -2)$, or (b) $(5, 15)$?

Which of the ordered pairs $(1, 7)$ and $(2, 9)$ are solution of the equation $y = 3x + 4$?

**Solution.** Substitute the points into the equation to determine which are solutions.

a) To determine if $(-3, -2)$ is a solution of $y = 2x + 5$, substitute $-3$ for $x$ and $-2$ for $y$ in the equation $y = 2x + 5$.

| | |
|---|---|
| $y = 2x + 5$ | Original equation. |
| $-2 = 2(-3) + 5$ | Substitute: $-3$ for $x$ and $-2$ for $y$. |
| $-2 = -6 + 5$ | Multiply first: $2(-3) = -6$ |
| $-2 = -1$ | Add: $-6 + 5 = -1$. |

Because the resulting statement is false, the ordered pair $(-3, -2)$ does **not** satisfy the equation. The ordered pair $(-3, -2)$ is **not** a solution of $y = 2x + 5$.

a) To determine if $(5, 15)$ is a solution of $y = 2x + 5$, substitute 5 for $x$ and 15 for $y$ in the equation $y = 2x + 5$.

| | |
|---|---|
| $y = 2x + 5$ | Original equation. |
| $15 = 2(5) + 5$ | Substitute: 5 for $x$ and 15 for $y$. |
| $15 = 10 + 5$ | Multiply first: $2(5) = 10$ |
| $15 = 15$ | Add: $10 + 5 = 15$. |

The resulting statement is true. The ordered pair $(5, 15)$ **does** satisfy the equation. Hence, $(5, 15)$ **is** a solution of $y = 2x + 5$.

Answer: $(1, 7)$

□

### The Graph of an Equation

We turn our attention to the *graph* of an equation.

**The Graph of an Equation.** The graph of an equation is the set of all ordered pairs that are solutions of the equation.

In the equation $y = 2x + 5$, the variable $y$ depends on the value of the variable $x$. For this reason, we call $y$ the *dependent* variable and $x$ the *independent* variable. We're free to make choices for $x$, but the value of $y$ will depend upon our choice for $x$.

We will also assign the horizontal axis to the independent variable $x$ and the vertical axis to the dependent variable $y$ (see Figure 8.7).

The graph of $y = 2x+5$ consists of all ordered pairs that are solutions of the equation $y = 2x+5$. So, our first task is to find ordered pairs that are solutions of $y = 2x + 5$. This is easily accomplished by selecting an arbitrary number of values, substituting them for $x$ in the equation $y = 2x + 5$, then calculating the resulting values of $y$. With this thought in mind, we pick arbitrary integers $-7, -6, \ldots, 2$, substitute them for $x$ in the equation $y = 2x + 5$, calculate the resulting value of $y$, and store the results in a table.

$$\begin{aligned}
y &= 2(-7) + 5 = -9\\
y &= 2(-6) + 5 = -7\\
y &= 2(-5) + 5 = -5\\
y &= 2(-4) + 5 = -3\\
y &= 2(-3) + 5 = -1\\
y &= 2(-2) + 5 = 1\\
y &= 2(-1) + 5 = 3\\
y &= 2(0) + 5 = 5\\
y &= 2(1) + 5 = 7\\
y &= 2(2) + 5 = 9
\end{aligned}$$

| $y = 2x + 5$ | | |
|---|---|---|
| $x$ | $y$ | $(x, y)$ |
| $-7$ | $-9$ | $(-7, -9)$ |
| $-6$ | $-7$ | $(-6, -7)$ |
| $-5$ | $-5$ | $(-5, -5)$ |
| $-4$ | $-3$ | $(-4, -3)$ |
| $-3$ | $-1$ | $(-3, -1)$ |
| $-2$ | $1$ | $(-2, 1)$ |
| $-1$ | $3$ | $(-1, 3)$ |
| $0$ | $5$ | $(0, 5)$ |
| $1$ | $7$ | $(1, 7)$ |
| $2$ | $9$ | $(2, 9)$ |

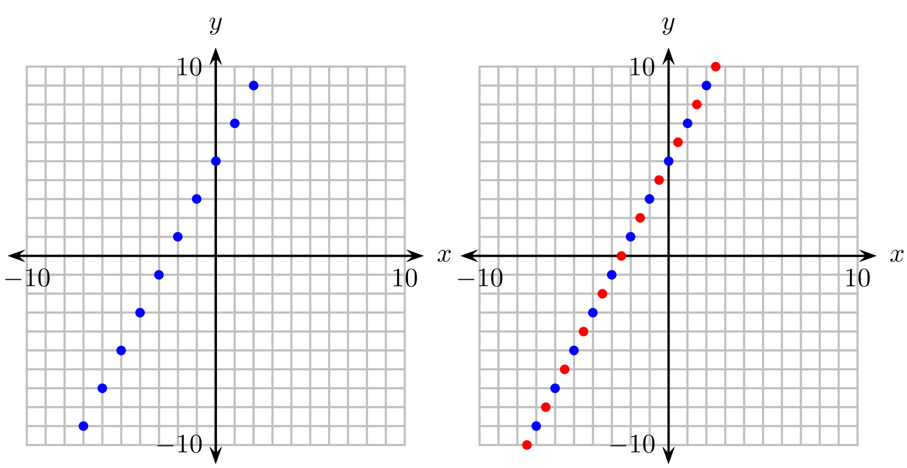
The result is 10 ordered pairs that satisfy the equation $y = 2x + 5$. Therefore, we have 10 ordered pairs that belong to the graph of $y = 2x + 5$. They are plotted in Figure 8.7(a).

However, we're not finished, because the graph of the equation $y = 2x + 5$ is the set of **all** points that satisfy the equation and we've only plotted 10 such points. Let's plot some more points. Select some more $x$-values, compute the corresponding $y$-value, and record the results in a table.

$$
\begin{aligned}
y &= 2(-7.5) + 5 = -10 \\
y &= 2(-6.5) + 5 = -8 \\
y &= 2(-5.5) + 5 = -6 \\
y &= 2(-4.5) + 5 = -4 \\
y &= 2(-3.5) + 5 = -2 \\
y &= 2(-2.5) + 5 = 0 \\
y &= 2(-1.5) + 5 = 2 \\
y &= 2(-0.5) + 5 = 4 \\
y &= 2(0.5) + 5 = 6 \\
y &= 2(1.5) + 5 = 8 \\
y &= 2(2.5) + 5 = 10
\end{aligned}
$$

| $y = 2x + 5$ | | |
|---|---|---|
| $x$ | $y$ | $(x, y)$ |
| $-7.5$ | $-10$ | $(-7.5, -10)$ |
| $-6.5$ | $-8$ | $(-6.5, -8)$ |
| $-5.5$ | $-6$ | $(-5.5, -6)$ |
| $-4.5$ | $-4$ | $(-4.5, -4)$ |
| $-3.5$ | $-2$ | $(-3.5, -2)$ |
| $-2.5$ | $0$ | $(-2.5, 0)$ |
| $-1.5$ | $2$ | $(-1.5, 2)$ |
| $-0.5$ | $4$ | $(-0.5, 4)$ |
| $0.5$ | $6$ | $(0.5, 6)$ |
| $1.5$ | $8$ | $(1.5, 8)$ |
| $2.5$ | $10$ | $(2.5, 10)$ |

That's 11 additional points that we add to the graph in Figure 8.7(b).




(a) Ten points that satisfy the equation $y = 2x + 5$.

(b) Eleven additional points that satisfy the equation $y = 2x + 5$.

Figure 8.7: Plotting points that satisfy the equation $y = 2x + 5$.

Note that we can continue indefinitely in this manner, adding points to the table and plotting them. However, sooner or later, we have to make a leap of faith, and imagine what the final graph will look like when all of the points that satisfy the equation $y = 2x + 5$ are plotted. We do so in Figure 8.8, where the final graph takes on the appearance of a line.

**Ruler Use.** All lines must be drawn with a ruler. This includes the $x$- and $y$-axes.

**Important Observation.** When we use a ruler to draw a line through the plotted points in Figure 8.7(b), arriving at the final result in Figure 8.8, we must understand that this is a shortcut technique for plotting all of the remaining ordered pairs that satisfy the equation. We're not really drawing a line through the plotted points. Rather, we're shading all of the ordered pairs that satisfy the equation $y = 2x + 5$.

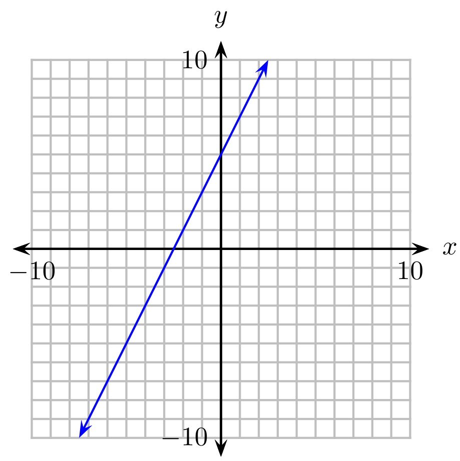



Figure 8.8: The graph of the equation $y = 2x + 5$.

**The Result.** The graph of the equation $y = 2x + 5$, pictured in Figure 8.8, is a line. Actually, the graph is an infinite collection of points satisfying the equation $y = 2x + 5$ that takes the shape of a line, but it's all right to say the graph of $y = 2x + 5$ is a line.

**Ordered Pairs and the Graph.** Because the graph of an equation is the collection of all ordered pairs that satisfy the equation, we have two important results:

1. If an ordered pair satisfies an equation, then the point in the Cartesian plane represented by the ordered pair is on the graph of the equation.

2. If a point is on the graph of an equation, then the ordered pair representation of that point satisfies the equation.

You Try It!

**EXAMPLE 2.** Find the value of $k$ so that the point $(2, k)$ is on the graph of the equation $y = 3x - 2$.

**Solution.** If the point $(2, k)$ is on the graph of $y = 3x - 2$, then it must satisfy the equation $y = 3x - 2$.

| | |
|---|---|
| $y = 3x - 2$ | Original equation. |
| $k = 3(2) - 2$ | The point $(2, k)$ is on the graph. Substitute 2 for $x$ and $k$ for $y$ in $y = 3x - 2$. |
| $k = 6 - 2$ | Multiply: $3(2) = 6$. |
| $k = 4$ | Subtract: $6 - 2 = 4$. |

Thus, $k = 4$.

Find the value of $k$ so that the point $(k, -3)$ is on the graph of the equation $y = 4x + 2$.

Answer: $k = -5/4$

□

## Linear Equations

Let's plot the graph of another equation.

You Try It!

**EXAMPLE 3.** Sketch the graph of $y = -2x + 1$.

**Solution.** Select arbitrary values of $x$: $-4$, $-3$, ..., 5. Substitute these values into the equation $y = -2x + 1$, calculate the resulting value of $y$, then arrange your results in a table.

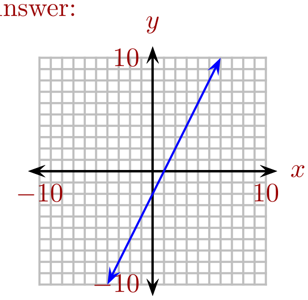
$$y = -2(-4) + 1 = 9$$
$$y = -2(-3) + 1 = 7$$
$$y = -2(-2) + 1 = 5$$
$$y = -2(-1) + 1 = 3$$
$$y = -2(0) + 1 = 1$$
$$y = -2(1) + 1 = -1$$
$$y = -2(2) + 1 = -3$$
$$y = -2(3) + 1 = -5$$
$$y = -2(4) + 1 = -7$$
$$y = -2(5) + 1 = -9$$

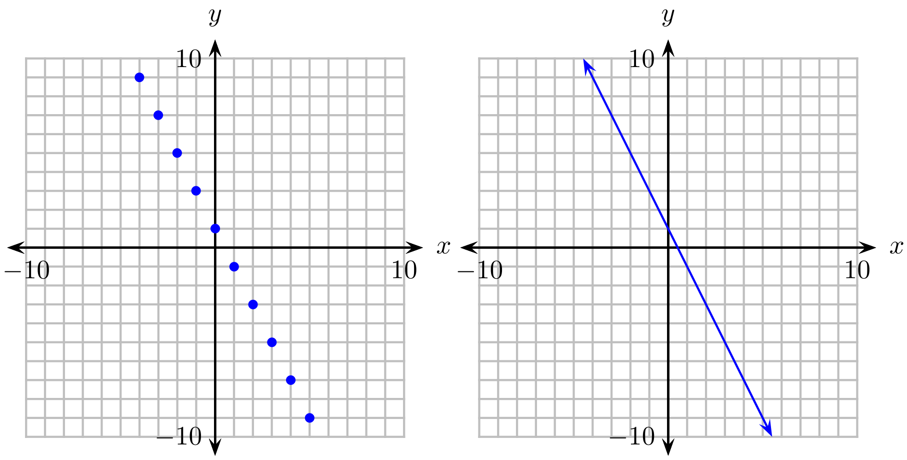
| $y = -2x + 1$ | | |
|---|---|---|
| $x$ | $y$ | $(x, y)$ |
| $-4$ | 9 | $(-4, 9)$ |
| $-3$ | 7 | $(-3, 7)$ |
| $-2$ | 5 | $(-2, 5)$ |
| $-1$ | 3 | $(-1, 3)$ |
| 0 | 1 | $(0, 1)$ |
| 1 | $-1$ | $(1, -1)$ |
| 2 | $-3$ | $(2, -3)$ |
| 3 | $-5$ | $(3, -5)$ |
| 4 | $-7$ | $(4, -7)$ |
| 5 | $-9$ | $(5, -9)$ |

Sketch the graph of $y = 2x - 2$.

We've plotted the points in the table in Figure 8.9(a). There is enough evidence in Figure 8.9(a) to imagine that if we plotted all of the points that satisfied the equation $y = -2x + 1$, the result would be the line shown in Figure 8.9(b).

Answer:




(a) Ten points that satisfy the equation $y = -2x + 1$.

(b) Plotting all points that satisfy the equation $y = -2x + 1$.

Figure 8.9: The graph of the equation $y = -2x + 1$ is a line.

The graph of $y = 2x + 5$ in Figure 8.8 is a line. The graph of $y = -2x + 1$ in Figure 8.9(b) is also a line. This would lead one to suspect that the graph of the equation $y = mx + b$, where $m$ and $b$ are constants, will always be a line. Indeed, this is always the case.

**Linear Equations.** The graph of $y = mx + b$, where $m$ and $b$ are constants, will always be a line. For this reason, the equation $y = mx + b$ is called a *linear equation.*

**You Try It!**

Which of the following equations is a linear equation?

a) $y = 2x^3 + 5$

b) $y = -3x - 5$

**EXAMPLE 4.** Which of the following equations is a linear equation? (a) $y = -3x + 4$, (b) $y = \frac{2}{3}x + 3$, and (c) $y = 2x^2 + 4$.

**Solution.** Compare each equation with the general form of a linear equation, $y = mx + b$.

a) Note that $y = -3x + 4$ has the form $y = mx + b$, where $m = -3$ and $b = 4$. Hence, $y = -3x + 4$ is a linear equation. Its graph is a line.

b) Note that $y = \frac{2}{3}x + 3$ has the form $y = mx + b$, where $m = 2/3$ and $b = 3$. Hence, $y = \frac{2}{3}x + 3$ is a linear equation. Its graph is a line.

c) The equation $y = 2x^2 + 4$ does **not** have the form $y = mx + b$. The exponent of 2 on the $x$ prevents this equation from being linear. This is a nonlinear equation. Its graph is not a line.

Answer: $y = -3x - 5$

□

The fact that $y = mx + b$ is a linear equation enables us to quickly sketch its graph.

**You Try It!**

**EXAMPLE 5.** Sketch the graph of $y = -\frac{3}{2}x + 4$.

Sketch the graph of $y = -\frac{1}{2}x + 2$.

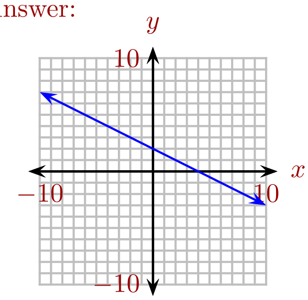
**Solution.** The equation $y = -\frac{3}{2}x + 4$ has the form $y = mx + b$. Therefore, the equation is linear and the graph will be a line. Because two points determine a line, we need only find two points that satisfy the equation $y = -\frac{3}{2}x + 4$, plot them, then draw a line through them with a ruler. We choose $x = -2$ and $x = 2$, calculate $y$, and record the results in a table.

$$y = -\frac{3}{2}(-2) + 4 = 3 + 4 = 7$$
$$y = -\frac{3}{2}(2) + 4 = -3 + 4 = 1$$

| $y = -\frac{3}{2}x + 4$ | | |
|---|---|---|
| $x$ | $y$ | $(x, y)$ |
| $-2$ | 7 | $(-2, 7)$ |
| 2 | 1 | $(2, 1)$ |

Plot the points $(-2, 7)$ and $(2, 1)$ and draw a line through them. The result is shown in Figure 8.10.

Answer:



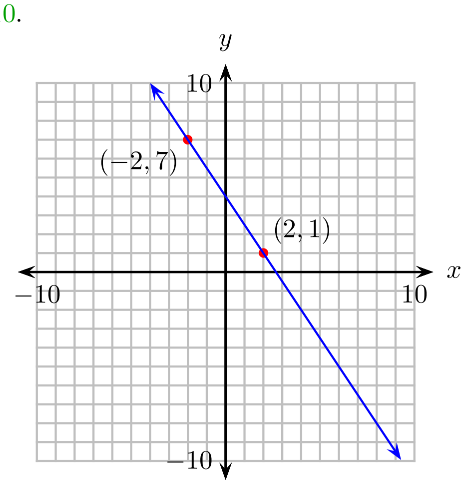




Figure 8.10: The graph of $y = -\frac{3}{2}x + 4$ is a line.

□

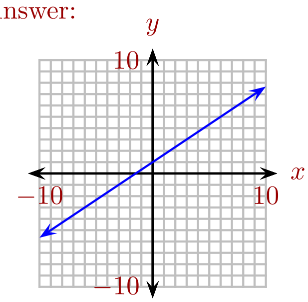
You may have noted in Example 5 that are choices of $-2$ and 2 for $x$ eased the calculation of the corresponding $y$-values because of the resulting cancellation.

**Choosing Strategic Values.** When plotting a linear equation, it is a good strategy to choose values of $x$ that simplify the calculation of the corresponding $y$-values.

**You Try It!**

Sketch the graph of $y = \frac{2}{3}x + 1$.

Answer:




**EXAMPLE 6.** Sketch the graph of $y = \frac{1}{3}x + 3$.

**Solution.** The equation $y = \frac{1}{3}x + 3$ has the form $y = mx + b$. Therefore, the equation is linear and the graph will be a line. Because two points determine a line, we need only find two points that satisfy the equation $y = \frac{1}{3}x + 3$, plot them, then draw a line through them with a ruler. We choose $x = -6$ and $x = 6$, calculate $y$, and record the results in a table.

$$y = \frac{1}{3}(-6) + 3 = -2 + 3 = 1$$
$$y = \frac{1}{3}(6) + 3 = 2 + 3 = 5$$

| $y = \frac{1}{3}x + 3$ | | |
|---|---|---|
| $x$ | $y$ | $(x, y)$ |
| $-6$ | 1 | $(-6, 1)$ |
| 6 | 5 | $(6, 5)$ |

Plot the points $(-6, 1)$ and $(6, 5)$ and draw a line through them. The result is shown in Figure 8.11.

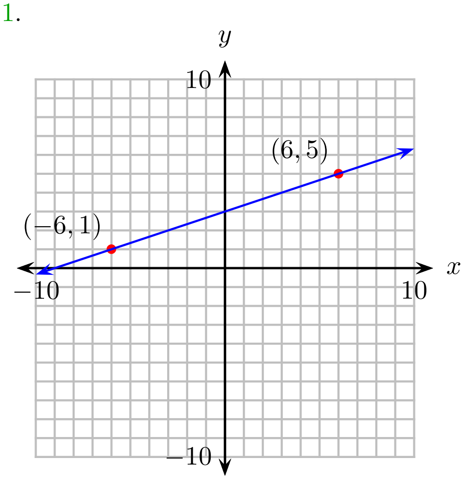



Figure 8.11: The graph of $y = \frac{1}{3}x + 3$ is a line.

□

## Exercises

**1**. Which of the points $(2,-14)$, $(-1,-6)$, $(-8,11)$, and $(3,-13)$ is a solution of the equation $y=-2x-8$?

**2**. Which of the points $(1,-2)$, $(8,23)$, $(-3,-23)$, and $(8,24)$ is a solution of the equation $y=4x-9$?

**3**. Which of the points $(1,-1)$, $(-2,20)$, $(-4,31)$, and $(-9,64)$ is a solution of the equation $y=-6x+7$?

**4**. Which of the points $(-8,-61)$, $(4,42)$, $(-3,-18)$, and $(-6,-46)$ is a solution of the equation $y=9x+8$?

**5**. Which of the points $(2,15)$, $(-8,-74)$, $(2,18)$, and $(5,40)$ is a solution of the equation $y=9x-3$?

**6**. Which of the points $(-9,-52)$, $(-8,-44)$, $(-7,-37)$, and $(8,35)$ is a solution of the equation $y=5x-5$?

**7**. Which of the points $(-2,12)$, $(-1,12)$, $(3,-10)$, and $(-2,14)$ is a solution of the equation $y=-5x+4$?

**8**. Which of the points $(6,25)$, $(-8,-14)$, $(8,33)$, and $(-7,-9)$ is a solution of the equation $y=3x+9$?

---

**9**. Determine $k$ so that the point $(9,k)$ is a solution of $y=-6x+1$.

**10**. Determine $k$ so that the point $(-9,k)$ is a solution of $y=2x+3$.

**11**. Determine $k$ so that the point $(k,7)$ is a solution of $y=-4x+1$.

**12**. Determine $k$ so that the point $(k,-4)$ is a solution of $y=8x+3$.

**13**. Determine $k$ so that the point $(k,1)$ is a solution of $y=4x+8$.

**14**. Determine $k$ so that the point $(k,-7)$ is a solution of $y=-7x+5$.

**15**. Determine $k$ so that the point $(-1,k)$ is a solution of $y=-5x+3$.

**16**. Determine $k$ so that the point $(-3,k)$ is a solution of $y=3x+3$.

---

In Exercises 17-24, which of the given equations is a linear equation?

**17**. $y=6x^2+4$, $y=x^2+6x+4$, $y=6x+4$, $y=\sqrt{6x+4}$

**18**. $y=-2x+1$, $y=x^2-2x+1$, $y=\sqrt{-2x+1}$, $y=-2x^2+1$

**19**. $y=x+7$, $y=\sqrt{x+7}$, $y=x^2+7$, $y=x^2+x+7$

**20**. $y=x^2+5x+1$, $y=5x^2+1$, $y=\sqrt{5x+1}$, $y=5x+1$

**21**. $y=x^2-2x-2$, $y=-2x^2-2$, $y=\sqrt{-2x-2}$, $y=-2x-2$

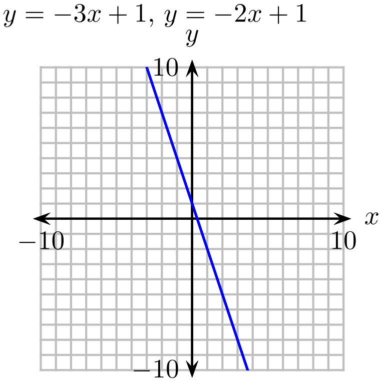
**22**. $y=x^2+5x-8$, $y=5x^2-8$, $y=\sqrt{5x-8}$, $y=5x-8$

**23**. $y=x^2+7x-3$, $y=7x^2-3$, $y=7x-3$, $y=\sqrt{7x-3}$

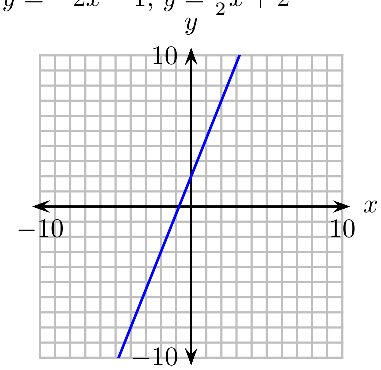
**24**. $y=\sqrt{-4x-3}$, $y=x^2-4x-3$, $y=-4x-3$, $y=-4x^2-3$

In Exercises 25-28, which of the given equations has the given graph?

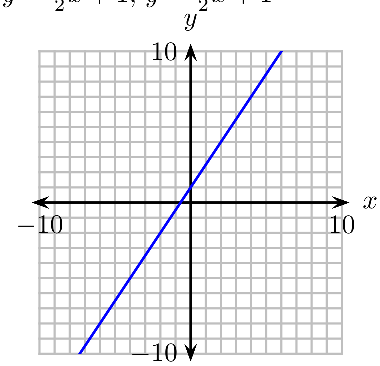
**25**. $y = -\frac{3}{2}x + 2$, $y = \frac{3}{2}x - 3$,
$y = -3x + 1$, $y = -2x + 1$



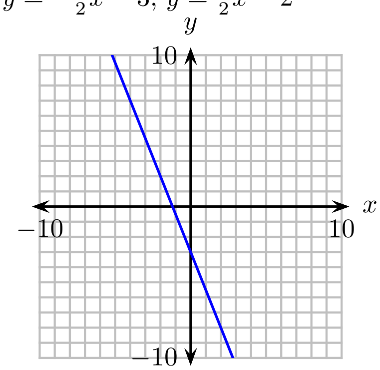

**26**. $y = -3x - 2$, $y = \frac{3}{2}x + 1$,
$y = -2x - 1$, $y = \frac{5}{2}x + 2$




**27**. $y = \frac{5}{2}x - 2$, $y = 3x + 3$,
$y = \frac{3}{2}x + 1$, $y = \frac{1}{2}x + 1$




**28**. $y = 3x + 1$, $y = \frac{5}{2}x - 1$,
$y = -\frac{5}{2}x - 3$, $y = \frac{3}{2}x - 2$




In Exercises 29-44, on graph paper, sketch the graph of the given equation.

**29**. $y = 3x - 2$

**30**. $y = \frac{5}{2}x + 1$

**31**. $y = -2x - 1$

**32**. $y = \frac{5}{2}x + 2$

**33**. $y = -2x + 2$

**34**. $y = -\frac{5}{2}x - 2$

**35**. $y = -2x - 2$

**36**. $y = -\frac{5}{2}x + 1$

**37**. $y = 2x - 2$

**38**. $y = \frac{5}{2}x - 1$

**39**. $y = \frac{3}{2}x + 1$

**40**. $y = 2x + 2$

**41**. $y = 2x - 3$

**42**. $y = -\frac{5}{2}x - 1$

**43**. $y = \frac{3}{2}x + 3$

**44**. $y = 3x + 1$

---

**45**. Sketch the lines $y = \frac{1}{2}x - 1$ and $y = \frac{5}{2}x - 2$ on graph paper. As you sweep your eyes from left to right, which line rises more quickly?

**46**. Sketch the lines $y = \frac{5}{2}x + 1$ and $y = 3x + 1$ on graph paper. As you sweep your eyes from left to right, which line rises more quickly?

**47**. Sketch the line $y = -\frac{1}{2}x + 1$ and $y = -3x + 3$. As you sweep your eyes from left to right, which line falls more quickly?

**48**. Sketch the line $y = -3x - 1$ and $y = -\frac{5}{2}x - 2$. As you sweep your eyes from left to right, which line falls more quickly?

**49**. Sketch the line $y = -3x - 1$ and $y = -\frac{1}{2}x - 2$. As you sweep your eyes from left to right, which line falls more quickly?

**50**. Sketch the line $y = -3x - 1$ and $y = -\frac{1}{2}x + 1$. As you sweep your eyes from left to right, which line falls more quickly?

**51**. Sketch the lines $y = \frac{3}{2}x - 2$ and $y = 3x + 1$ on graph paper. As you sweep your eyes from left to right, which line rises more quickly?

**52**. Sketch the lines $y = \frac{1}{2}x + 3$ and $y = \frac{5}{2}x + 1$ on graph paper. As you sweep your eyes from left to right, which line rises more quickly?

---

## ❧ ❧ ❧ Answers ☙ ☙ ☙

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**1.** $(-1, -6)$

**3.** $(-4, 31)$

**5.** $(2, 15)$

**7.** $(-2, 14)$

**9.** $k = -53$

**11.** $k = -\dfrac{3}{2}$

**13.** $k = -\dfrac{7}{4}$

**15.** $k = 8$

**17.** $y = 6x + 4$

**19.** $y = x + 7$

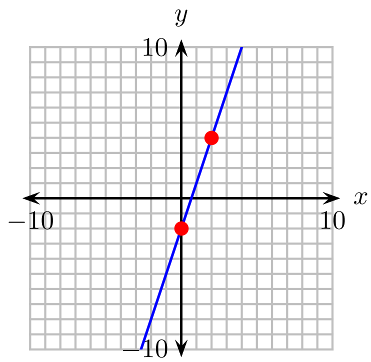
**21.** $y = -2x - 2$

**23.** $y = 7x - 3$

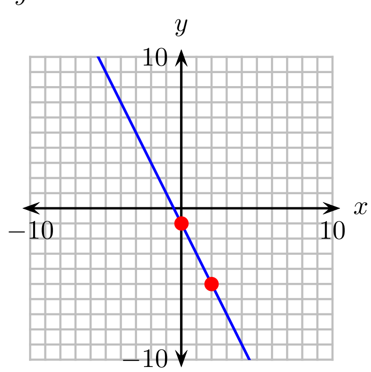
**25.** $y = -3x + 1$

**27.** $y = \frac{3}{2}x + 1$

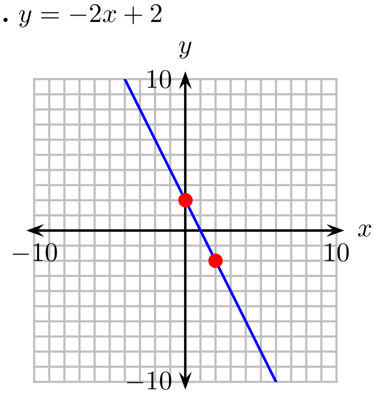
**29.** $y = 3x - 2$



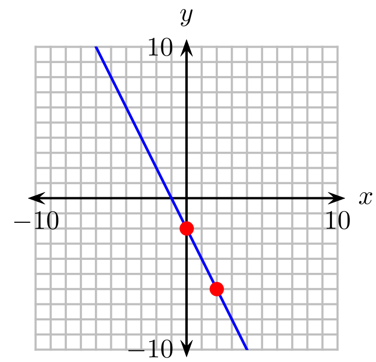

**31.** $y = -2x - 1$



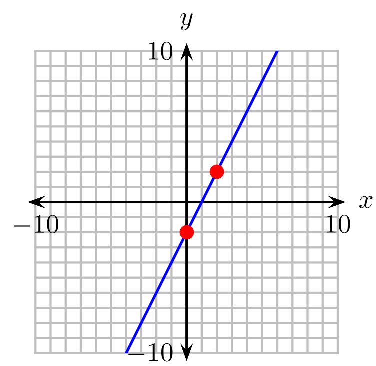

**33.** $y = -2x + 2$



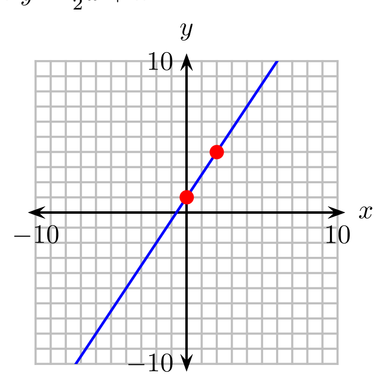

**35.** $y = -2x - 2$



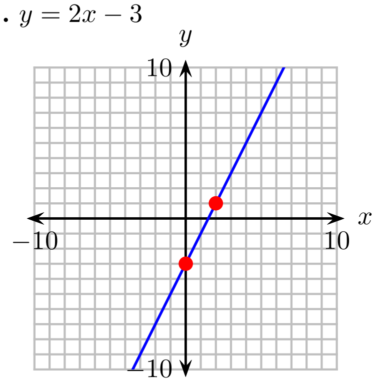

**37.** $y = 2x - 2$



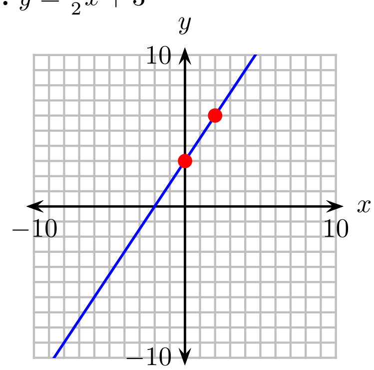

**39.** $y = \frac{3}{2}x + 1$



**41.** $y = 2x - 3$




**43.** $y = \frac{3}{2}x + 3$




**45.** The graph of $y = \frac{5}{2}x - 2$ rises more quickly.

**47.** The graph of $y = -3x + 3$ falls more quickly.

**49.** The graph of $y = -3x - 1$ falls more quickly.

**51.** The graph of $y = 3x + 1$ rises more quickly.

# Index